This is a first course in commutative algebra. It will be geared toward the potential AG applications, but it can also be a standalone course for those interested in CA in its own right.

Background needed: Roughly the material of a graduate or advanced undergraduate course. Point-set topology is useful as well.

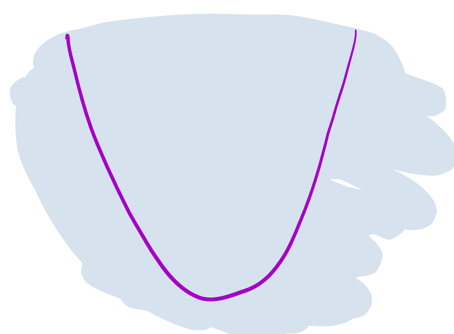
Material covered: We will be loosely following Mel Hochster's notes (see link on website), but we will fill in gaps and use problems from Atiyah-Macdonald and Eisenbud.

Motivation

Commutative algebra is intimately connected to algebraic geometry because the algebraic properties of a commutative ring are reflected in geometric properties of the corresponding geometric object (e.g. variety, scheme) and vice versa.

A very geometric collection of examples in CA are polynomial rings over a field $k=\overline{k}$. i.e. $R=k[x_1,\dots,x_n]$.

In this case, the zero set of a polynomial, say $x_1^2-x_2$ is a locus in k^n ($=\mathbb{A}^n$, "affine n-space")



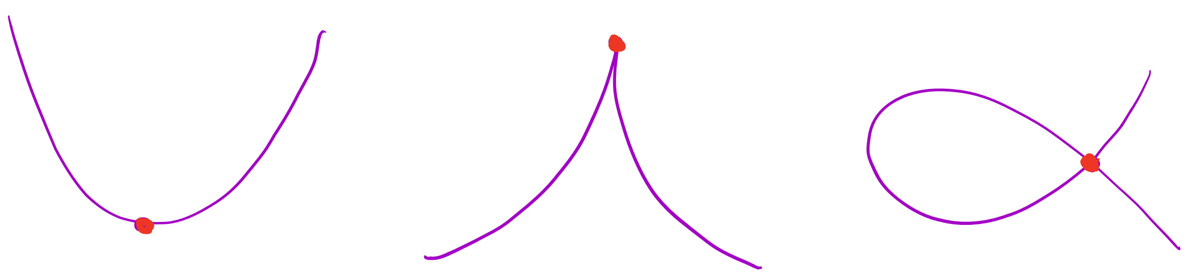
And in fact, the prime ideals in $k[x_1, \dots, x_n]$ correspond to "subvarieties" of affine space, called "affine varieties".

More generally, the prime ideals of an arbitrary ring R correspond to the points of the "scheme" corresponding to R, called an "affine scheme".

All varieties and schemes can be constructed by gluing together affine varieties / schemes, so, in a way, commutative algebra can be thought of as "local algebraic geometry".

Local rings also have a geometric interpretation: they can describe the geometry of a scheme near a point.

e.g.



a smooth point, cusp, and node all appear to be different geometrically, and, indeed, this is reflected in their corresponding "local rings".

We will make this all more precise soon.